Cantilever Brake Geometry: Setup and Mechanical Advantage

02/28/10
Benno Belhumeur
bbelhumeur@gmail.com

Background:

In setting up my cantilever brakes I closely read both Sheldon Brown's articles on cantilever geometry[1] and setup as well as the Bicycle Quarterly articles in the issue on brakes.[2] I found that both articles offered sound advice on how to manage the many variables involved in brake setup. Neither article offered a quantitative analysis, however, nor did they attempt to examine the interplay between the various setup variables. I wanted to be able to not only quantitatively analyze the setup of cantilevers, but also to isolate individual variables to see the relationships between all the variables. To do so I developed an equation that could be plotted to show mechanical advantage as a function of either yoke height or cantilever angle, including constant values for stud position and arm length. With this you can compare setups of an individual brake or compare any two brakes to see their relative mechanical advantage and the ideal setups for each.

Analysis:

Bicycle brake systems are simple mechanisms which magnify the force of the rider squeezing the brake lever and transmit it to the pads which squeeze the rim or disc to slow the bicycle. We can say that any brake system which multiplies the force applied to the lever has a "mechanical advantage." Mechanical advantage can be defined as "the factor by which a mechanism multiplies the force or torque applied to it."[3] In the case of brakes, the rider squeezes the lever with a certain force which is amplified by the brake and lever mechanisms to result in the pads squeezing the rim with a greater force. The mechanical advantage can be represented as the resulting force at the rim divided by the force applied at the lever. For example, if the force applied to the rim is eight times greater than that applied by the rider at the lever, we would say that entire brake system, including brake and lever, has a mechanical advantage of eight.

Typically, both the brake lever and the brake itself increase mechanical advantage of the brake system. In this article, I am concerned solely with the increase in mechanical advantage of the brake itself and only one type of brake, the cantilever. Additionally, I only analyze the theoretical mechanical advantage of the system. Losses due to cable friction and flex in the system, whether in the brake or lever, or due to poorly installed housing, can have a significant impact on your brake's performance. Sheldon Brown's articles provide excellent guidance on general brake setup, both theoretically and mechanically.

Figure 1[4] shows a typical cantilever setup. The line through *YO* represents the center line of the bike. The line through *BO* represents a level line through the pivot and the line *ZP* represents a plumb line through the pivot. The angle α represents the cantilever angle[5], the angle γ represents the yoke angle and the angle β, the anchor angle. The distance *YO* is the yoke height, the distance *PA* is the arm length, *YA* is the straddle cable length, and *PO* the pivot width.

1 The Geometry of Cantilever Brakes, Sheldon Brown (http://www.sheldonbrown.com/cantilever-geometry.html)
2 Bicycle Quarterly (Vol. 7, No. 2), Vintage Bicycle Press
3 Mechanical Advantage, Wikipedia, http://en.wikipedia.org/wiki/Mechanical_advantage
4 Background image by Sheldon Brown
5 Brown refers to cantilever angle as being the angle between line *AP* and line *RP*. This is a useful way to define it in order to allow people to visually discern the rough design of an individual brake. It is less useful for the purpose of geometrically analyzing the setup of brakes. This angle can be divided into two smaller angles. The angle between lines *ZP* and *RP* is relatively constant, at least on any given bike and even on different bikes doesn't change that much. For our analysis, we will treat the angle between *AP* and *ZP* as the cantilever angle.

The mechanical advantage is altered by three main factors[6] :

- The yoke angle: MA is inversely proportional to sin γ.

- The effective arm length: MA is proportional to the effective arm length, which is a perpendicular drawn from the pivot to the straddle cable equal to sin $\beta \cdot PA$.

- The effective pad arm length: MA is inversely proportional to the effective pad arm length, which is equal to the line *RX*, or *DO*[7].

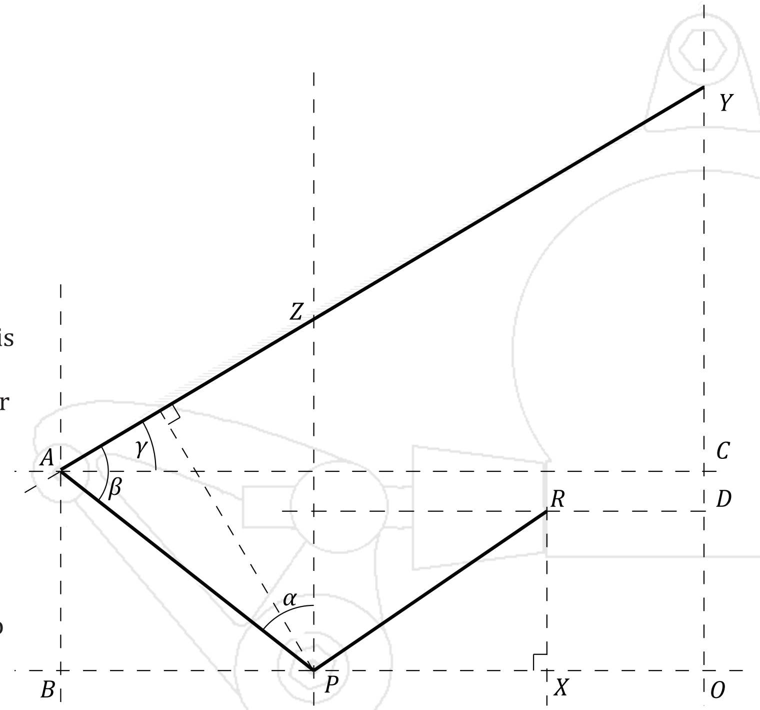
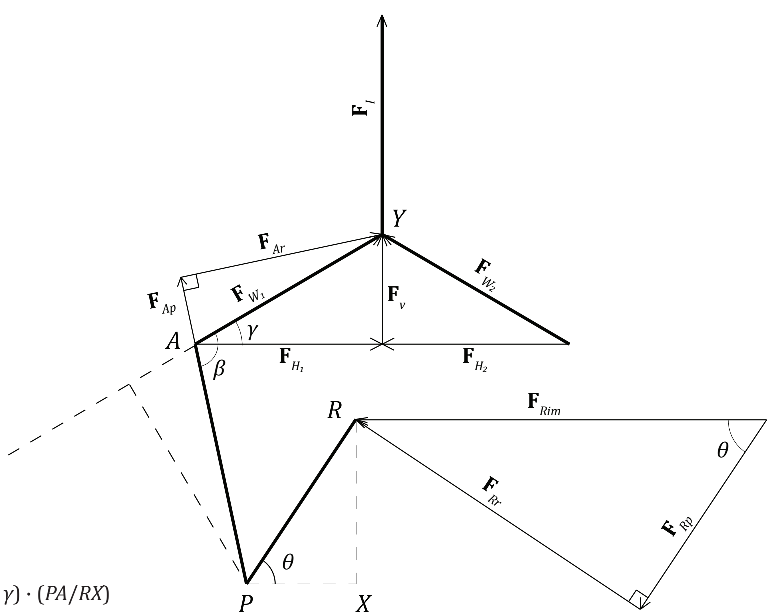


Figure 1: Typical Cantilever Geometry

Combining these we are left with:

$MA = (1/\sin\gamma) \cdot \sin\beta \cdot (PA\ /\ DO)$

We can now reformulate this equation into terms that are more useful to us. We can replace β and γ with variables that represent the pivot locations, cantilever angle, yoke height and straddle cable length. These variables are more representative of physical features of your bike and brakes and of things that you might adjust, than are β and γ. Additionally, by doing so we'll be able to plot MA as a function of either yoke height or cantilever angle, and have the ability to modify the other variables individually and see the results.



---

6 Analysis:

$\mathbf{F}_I = \mathbf{F}_{W1} + \mathbf{F}_{W2}$
$\mathbf{F}_W = \mathbf{F}_V + \mathbf{F}_H$
$\mathbf{F}_{H1} = -\mathbf{F}_{H2}$
$\mathbf{F}_I = \mathbf{F}_V + \mathbf{F}_{H1} + \mathbf{F}_V + \mathbf{F}_{H2} = \mathbf{F}_V + \mathbf{F}_V$
$F_V = \sin\gamma \cdot F_{w1}$
$F_I = 2\sin\gamma \cdot F_{w1}$
$F_{w1} = F_I / (2\sin\gamma)$
F_{Ar} = force perpendicular to *PA* at *A*
$F_{Ar} = \sin\beta \cdot F_{w1} = \sin\beta \cdot (F_I / (2\sin\gamma))$
F_{Rr} = force perpendicular to *PR* at *R*

Brake is in equilibrium so:
$F_{Ar} \cdot PA = F_{Rr} \cdot PR$
$F_{Rr} = \sin\theta \cdot F_{Rim}$
$F_{Ar} \cdot PA = \sin\theta \cdot F_{Rim} \cdot PR$
$\sin\beta \cdot (F_I / (2\sin\gamma)) \cdot PA = \sin\theta \cdot F_{Rim} \cdot PR$
$F_I \cdot (\sin\beta / (2\sin\gamma)) \cdot PA = F_{Rim} \cdot \sin\theta \cdot PR$
$F_I \cdot (\sin\beta / (2\sin\gamma)) \cdot PA = F_{Rim} \cdot (RX/PR) \cdot PR$
$F_I \cdot (\sin\beta / (2\sin\gamma)) \cdot PA = F_{Rim} \cdot RX$
$F_{Rim} = F_I \cdot (\sin\beta / (2\sin\gamma)) \cdot (PA/RX)$

Force on both sides of rim $= 2 \cdot (F_{Rim}) = F_I \cdot (\sin\beta / \sin\gamma) \cdot (PA/RX)$
$MA = (\sin\beta / \sin\gamma) \cdot (PA/RX)$

7 Brown states that the MA is inversely proportional to the length PR (not DO). This would be true if PR were striking the rim squarely. Since the pad arm is hitting the rim obliquely, it is the vertical component DO, which is parallel to the face of the rim and perpendicular to the applied force, that is relevant to MA.

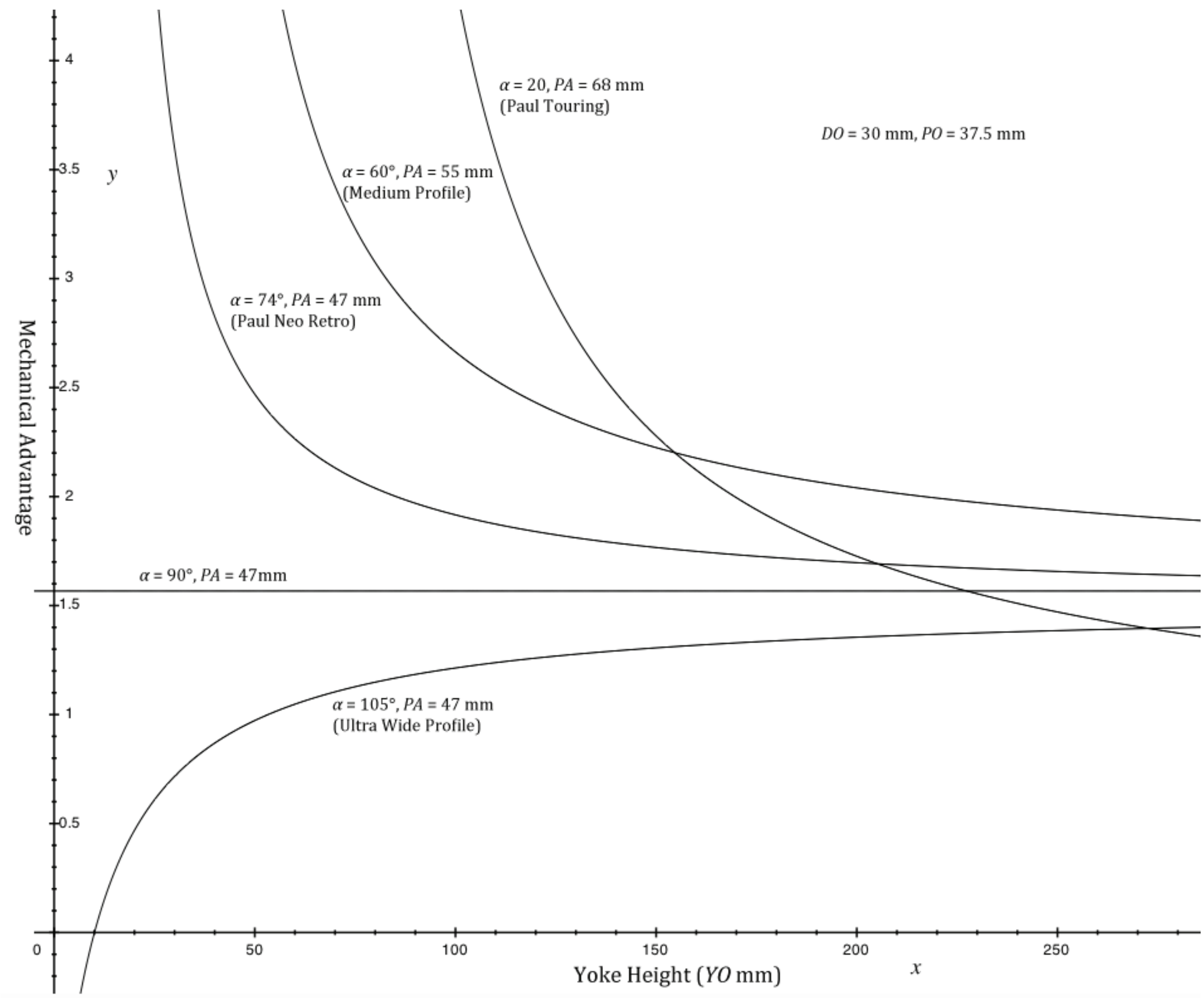


Figure 2: Mechanical Advantage as a Function of Yoke Height

In Figure 2[8] you can see the mechanical advantage of five brakes (two real, three hypothetical) plotted as a function of yoke height (*YO*). In this example, I have measured my Thorn Raven Tour to provide the constants *DO* and *PO*. Most bikes will have similar values.

The brakes are:

1) the Paul Touring, a low-ish[9] profile brake with a long arm, which is made possible by its more vertical orientation; on a wide profile brake an arm this long would stick out too far.
2) a hypothetical medium profile brake with a medium angle and an arm length between the low profile and the wide profile lengths.
3) the Paul Neo Retro, a wide profile brake similar in design to the Mafac cantilever

8 $BP = \sin\alpha \cdot PA$
$CO = AB = \cos\alpha \cdot PA$
$AC = BP + PO = (\sin\alpha \cdot PA) + PO$
$YC = YO - CO = YO - AB = YO - (\cos\alpha \cdot PA)$
$\tan\gamma = YC/AC = (YO - (\cos\alpha \cdot PA))/((\sin\alpha \cdot PA) + PO)$
$\gamma = \tan^{-1}((YO - (\cos\alpha \cdot PA))/((\sin\alpha \cdot PA) + PO))$
$\beta = \gamma + (90 - \alpha) = \tan^{-1}((YO - (\cos\alpha \cdot PA))/((\sin\alpha \cdot PA) + PO)) + (90 - \alpha)$

Thus we have:

$MA = (1/\sin\gamma) \cdot \sin\beta \cdot (PA / DO) = 1/\sin(\tan^{-1}((YO - (\cos\alpha \cdot PA))/((\sin\alpha \cdot PA) + PO))) \cdot \sin(\tan^{-1}((YO - (\cos\alpha \cdot PA))/((\sin\alpha \cdot PA) + PO)) + (90 - \alpha)) \cdot PA / DO$

9 Many 1990’s era boutique cantis appear to have cantilever angles close to zero degrees. See bikepro.com.

4) a hypothetical brake with a 90 degree cantilever angle
5) a hypothetical brake with an ultra wide angle, greater than 90 degrees

You can see by looking at the graph that the further the cantilever angle is from 90 degrees, the more radical the change in MA is for varying yoke heights. Generally, the steeper the curve, the less flexibility you have when setting the brakes up.

Another thing that jumps out of this graph is that for brakes with anchors higher than the pivots, in other words with cantilever angles less than 90 degrees, lowering the yoke *always* results in higher MA, for better or worse. There were times in setting up wide profile brakes where I was unsure whether it was more important to make the straddle cable perpendicular to the brake arm or to lower the yoke. The answer is that if you are searching for more MA, lower the yoke[10].

Additionally you can see the result of a brake with a 90 degree cantilever angle. It is a brake with constant mechanical advantage for all yoke heights. A very easy setup.

Looking at the example where the cantilever angle is greater than 90 degrees, you can see that here is the one situation where raising the yoke can actually increase the MA. In looking at many of the old French bikes and Daniel Rebour drawings you often see cantis with super wide angles, and while it is tough to tell the geometry of the brakes at the rim from photos and drawings, it certainly appears that the cantilever angle is greater than 90 degrees on some of them.

Looking at Figure 2, you can see that this generally results in a low MA, although one that should increase in MA very nicely as the brake approaches the rim. By adding a main brake cable pulley above the yoke, as you often see on these old bikes, MA could be effectively doubled.

It certainly appears that the ultra wide cantilever will have the effect of increasing in MA as they swing through their arc. Indeed it is tempting to read the graph as representative of this swing, as while the brake is being pulled the yoke height is increasing, but the graph is really for a static angle α. That is to say, the graph really represents how changing the length of the straddle cable and thus the yoke height for any given brake will affect its MA at the rim.

We may, however, wish to see how the MA of a brake changes as it swings through its arc. For example, we may want to know whether to push the pads in or out on their studs if we have a fixed length straddle wire. Also, as the pads wear the effective cantilever angle decreases and the yoke height increases when the pads touch the rim, as they have farther to travel. In Figure 2, we were examining yoke height, but changing the straddle cable length to change the height of the yoke. Now we want to keep the straddle cable length the same and see how the MA varies as the yoke is pulled up by the lever and the arm swings.

In Figure 3[11], each line represents a given straddle cable length/arm length combination. This represents an individual setup of a brake. We can now look at the MA as a function of the angle α (and implicitly the changing yoke height as the lever is pulled). The *y* axis represents the MA and the *x* axis represents angle α as the brake swings through it's arc (read from right to left).

10 This does not mean that you should not attempt to maximize the effective arm length by getting it as close to perpendicular to the straddle cable as you can by pushing the brake pads all the way in on the arms for wide angle brakes, in other words changing the α; just that once you have determined the best setup for the brake pads, maximum MA is gained by lowering the straddle cable, even if it shortens the effective arm length, though again this only applies to cantilever angles less than 90 degrees.

11
$$\gamma = \cos^{-1}(AC/YA)$$
$$AC = PO + (\sin\alpha \cdot PA)$$
$$\gamma = \cos^{-1}((PO + (\sin\alpha \cdot PA))/YA)$$
$$\beta = \gamma + 90 - \alpha$$
$$\beta = \cos^{-1}((PO + (\sin\alpha \cdot PA))/YA) + 90 - \alpha$$

Thus we have:
$$y = MA = (1/\sin\gamma) \cdot \sin\beta \cdot (PA / DO) = 1/\sin(\cos^{-1}((PO + (\sin\alpha \cdot PA))/YA)) \cdot \sin(\cos^{-1}((PO + (\sin\alpha \cdot PA))/YA) + 90 - \alpha) \cdot PA/DO$$

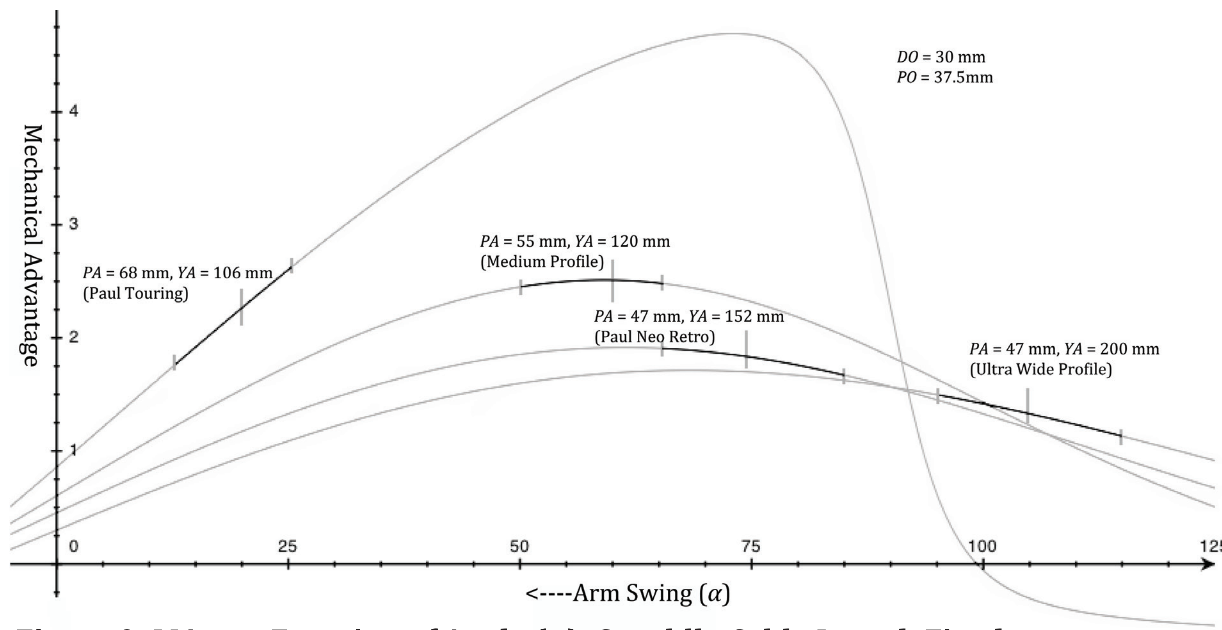


Figure 3: MA as a Function of Angle (α): Straddle Cable Length Fixed

For this figure, I have plotted the same brakes, dropping only the 90 degree cantilever. I've chosen a straddle cable length (*YA*) based on how these brakes are set up on my bike, or would likely be set up based on the MA curves shown in Figure 2. The line representing the Paul Touring is based on *YA* of 106 mm, which is how it is set up on the rear of my Thorn Raven Tour. This is about the shortest cable I can have with 2.0 tires and fenders. The faint gray line shows the hypothetical MA if the brake could swing through its entire arc. The darker line shows the more likely angle through which the brake will actually swing. There is a small tick on the curve where the pad will make contact with the rim on average. For example, the tick for the Paul Touring brakes appears above 20 degrees on the *x* axis, which is about where it makes contact with the rim on my bike, but with different pads or rims or pivot locations it could swing below 15 degrees.

The graph shows a MA of about 2.3 for the brake at this point in its swing. Notice how steeply the MA is declining however as the pads approach the rim. You can see that these brakes would experience a significant drop in MA as the brake pads wore and they touched the rim a few degrees later in the swing. This is not true of the other three setups however which have increasing or only slightly declining MA as they swing through their arc.

It is important to remember that this is not completely due to the angle of the brake, as it is the straddle cable length and arm length combined that create the shape of the curve. It is the combination of the long arm length and short straddle cable that makes the "hump" so pronounced for the Paul Touring, but the brake's cantilever angle determines where on the curve the swing of the brake takes place. You can see then, how lower profile cantilevers can have radically declining MA as their cantilever angle causes them to be built with longer arms to make up for the bad angle and set up with lower straddle cables to further increase MA. Both factors conspire to make it difficult to achieve to right amount of MA with low profiles.

In this case however, since the Paul's are not extremely low profile, I could just take small reduction in MA by increasing the yoke height (*YO*). Currently the straddle length (*YA*) of 106 mm translates into a yoke height (*YO*) of about 150 mm. Increasing this to 200 mm would reduce the MA of the brake close to that of the Neo Retro (see Figure 2), but also flatten out the above curve making its MA a little bit more linear for different rim and pad conditions. Or I could just leave it, as on my bike set up as it is, it has more MA than the Neo Retro under all conditions.

You can also see that the medium profile brake set up with *YA* = 120 mm, which translates into roughly a 120 mm yoke height (*YO*). This is a bit lower than the setup of the Paul Touring, but it is required to get similar MA. Though this won't clear quite as much tire/fender as either Paul, this setup has not only a pretty constant MA through its entire swing, but it is generally as powerful or more powerful than the Paul Touring, with a significantly shorter arm, though it sticks out further. Additionally, it is substantially more powerful than the wide profile brakes throughout its entire range, due mostly to it's longer arm.

Summary:

I think that what stands out the most here is the fact that lowering the straddle cable always increases MA, for brakes with cantilever angles less than 90 degrees. Knowing this removes a lot of doubt when adjusting your brakes when you take into account the other variables in setup. It is important to be able to recognize when your brakes have too little or too much MA, though, in order for this to be useful. Brakes with too little MA often feel great as they are firm to the squeeze, whereas a brake with enough MA will feel slightly spongy from brake pad compression. On a brake with too much MA, the lever will either bottom out before the pads hit the rim, or the pads will have to be set so close to the rim that they hit the rim if the brake goes slightly out of adjustment or the wheels go slightly out of true.

Also, there is little reason to put much stock in the oft heard rule of thumb that the brake arm should intersect with the straddle cable at a right angle. This may occasionally be true in practice, as the actual arms of cantilever brakes are usually bent, whereas in the geometric analysis we are examining the theoretical arm (*PA*) which is always straight. Perhaps some manufacturers bend the arm at an angle such that the ideal yoke setting creates a perpendicular intersection at the physical arm. It's clear, though, that a straddle cable perpendicular to the theoretical arm is not the best setup for most brakes.

On low profile brakes, this will create too much MA with rapidly declining MA as the brakes swing through their arc. On wide profile brakes, this can never truly be achieved as the yoke height would be extremely high. Even if it could, there would be no reason to set the yoke that high. Greatest MA is achieved by setting the yoke as low as possible while still clearing any obstructions like fat tires, fenders or light mounts. These brakes are generally low enough in MA that even with a low yoke height they will still be capable of being set up with the pads a good distance from the rim for mud clearance.

Finally, for medium profile brakes, you probably can set the cable perpendicular to *PA*, but these brakes are so forgiving, that just about any other setting of the yoke will work too.

Other than that, I think the data confirm the conventional wisdom that low profile brakes can be powerful, but tricky to adjust. Additionally they go out of adjustment easily with pad wear. Medium profile brakes have a combination of good MA with a wide range of yoke settings and wide profile brakes have lower MA with good clearance and virtually foolproof setup.

I've written a online visual calculator that allows you to see how changes to cantilever brake variables affect mechanical advantage in real time. Go to http://www.circleacycles.com/cantilevers/ for more info.